UE Fluids Problem 2 Solution Fall 08

1a)

$$w(x) = w_\infty + \frac{1}{2\pi}\int_0^c \gamma(\xi)\,\frac{d\xi}{\xi - x} = V_\infty \underbrace{\sin\alpha}_{\simeq \alpha} + \frac{2V_\infty\alpha}{2\pi}\int_0^c \sqrt{\frac{c-\xi}{\xi}}\,\frac{d\xi}{\xi - x}$$

At $x = -0.5$, $\quad w \simeq V_\infty \alpha\left[1 + \frac{1}{\pi}\int_0^1 \sqrt{\frac{1-\xi}{\xi}}\,\frac{d\xi}{\xi + 0.5}\right]$

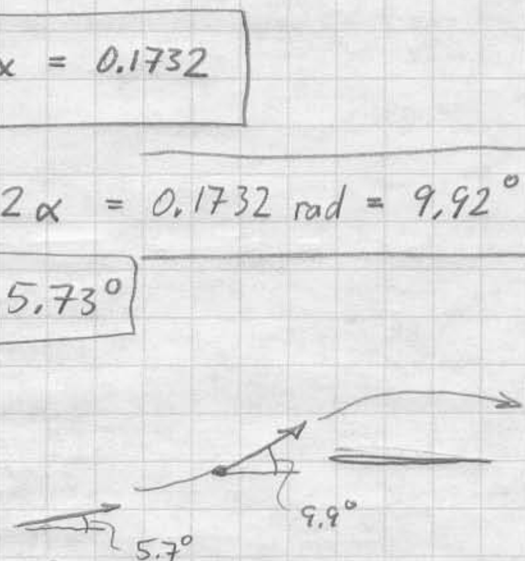
Numerically, $\int_0^1 \sqrt{\frac{1-\xi}{\xi}}\,\frac{d\xi}{\xi+0.5} = 2.30$

$$\therefore \boxed{w = V_\infty\alpha\left[1 + \frac{2.30}{\pi}\right] = 1.732\,V_\infty\alpha = 0.1732}$$

1b) Flow angle $\boxed{\theta = \arctan\frac{w}{u} \simeq \frac{w}{V_\infty} = 1.732\,\alpha = 0.1732 \text{ rad} = 9.92^\circ}$

Freestream angle $= \boxed{\alpha = 0.10 \text{ rad} = 5.73^\circ}$

Airfoil causes upwash ahead of it.



1c) A sensor at $x = -0.5$ will see $\theta = 9.9^\circ$, not $\alpha = 5.7^\circ$.

It's difficult to measure α because the airfoil itself changes the local flow angles.

Note for 1a).

The integrand has (an integrable) singularity $\sim \frac{1}{\sqrt{\xi}}$ at $\xi = 0$

This requires lots of points in the numerical integration.

One good technique is to "remove" the singularity from numerical integration.

$$\lim_{\xi\to 0} f(\xi) \equiv \sqrt{\frac{1-\xi}{\xi}}\,\frac{1}{\xi+0.5} = \lim_{\xi\to 0}\sqrt{\frac{1-0}{\xi}}\,\frac{1}{0+0.5} = \frac{2}{\sqrt{\xi}} \equiv g(\xi)$$

$$I = \int_0^1 f(\xi)\,d\xi = \underbrace{\int_0^1 [f(\xi) - g(\xi)]\,d\xi}_{\text{Do numerically } = -1.70} + \underbrace{\int_0^1 g(\xi)\,d\xi}_{\text{Do analytically } = \int_0^1 \frac{2}{\sqrt{\xi}}d\xi = 4\sqrt{\xi}\big|_0^1 = 4}$$

; note $f(\xi) - g(\xi)$ is well-behaved

$I = -1.7 + 4 = 2.30$